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CP-sensitive observables in chargino production with transverse $e^{\pm}$ beam polarization

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Abstract. We consider the process $e^+e^- \to \tilde{\chi}_i^+\tilde{\chi}_j^-$ at a linear collider with transverse $e^{\pm}$ beam polarization. We investigate the influence of the CP phases on azimuthal asymmetries in $e^+e^- \to \tilde{\chi}_i^+\tilde{\chi}_j^-$ with subsequent two-body decays $\tilde{\chi}_j^- \to \tilde{\nu}_\ell \ell^-$ and $\tilde{\chi}_j^- \to W^-\tilde{\chi}_1^0$. We show that triple product correlations involving the transverse $e^{\pm}$ beam polarization vanish if at least one subsequent chargino decay is not observed. We derive this result within the minimal supersymmetric standard model (MSSM) with complex parameters; however, it holds also in the general MSSM with SUSY flavor violation.

1 Introduction

Supersymmetry (SUSY) is at present the most studied extension of the standard model (SM) [1]. Some of the SUSY parameters may be complex and are potential new sources of CP-violation [2,3]. In the chargino sector of the minimal supersymmetric standard model (MSSM) the higgsino mass parameter μ can be complex, while the $SU(2)$ gaugino mass parameter M_2 can be chosen real by redefining the fields. In the neutralino sector of the MSSM also the gaugino mass parameter M_1 can be complex. The precise determination of the underlying SUSY parameters will be one of the main goals of a future e^+e^- linear collider (LC) with high luminosity [4]. The phases $\varphi_\mu, \varphi_{M_1}$ will give rise to CP-odd observables which may also be measured in future collider experiments.

The study of chargino production

$$e^+e^- \to \tilde{\chi}_i^+\tilde{\chi}_j^- \ , \qquad i,j = 1,2 \ , \tag{1}$$

will play an important role at the LC. This process has been studied quite often in the literature [5–7]. In [6] a method has been developed to determine the underlying parameters $M_2, |\mu|, \tan\beta$, including $\cos\varphi_\mu$, by measurements of the chargino masses and cross sections. The formulae for the cross section of (1) including longitudinal and transverse beam polarizations have also been given and azimuthal asymmetries have been proposed in [7,8].

In principle experiments with transverse $e^{\pm}$ beam polarization may offer the possibility of precision studies of the effects of CP-violation and new physics. For example, it has been shown that in the reactions $e^+e^- \to W^+W^-$ [9], $e^+e^- \to f\ \bar{f}$ [10] and $e^+e^- \to t\ \bar{t}$ [11] transverse $e^{\pm}$ beam polarization is indeed very helpful to disentangle effects of new physics. It is, therefore, tempting to study the potential of transverse beam polarization for measuring CP-sensitive observables also in chargino production (1). Triple products give rise to T-odd observables which may be useful to measure the CP phases involved. When only the cross sections of (1) are measured, summed over the polarizations of the produced charginos, one may try the following triple products involving the transverse beam polarization:

$$O_{\mathrm{T}}^1 = (\mathbf{p}_e \times \mathbf{p}_{\tilde{\chi}}) \cdot \mathbf{t}^{\pm} \ , \qquad O_{\mathrm{T}}^2 = (\mathbf{t}^+ \times \mathbf{t}^-) \cdot \mathbf{p}_{\chi} \ , \tag{2}$$

where $\mathbf{t}^-$ ($\mathbf{t}^+$) is the 3-vector of the transverse polarization of the e^- (e^+), and $\mathbf{p}_e$ and $\mathbf{p}_{\tilde{\chi}}$ are the momentum vectors of e^- (or e^+) and $\tilde{\chi}_i^{\pm}$. The leading contribution (at tree-level) to such a term in the matrix element would be solely due to CP-violation. However, from the formulae given in [7] it can be seen that terms involving the triple products $O_{\mathrm{T}}^{1,2}$ vanish if only chargino production cross sections are measured. This follows also from the general analysis in [11] and [12]. As a next step one may try to use triple products which involve also the subsequent decay of one of the two charginos. For definiteness one may consider the two-body decays

$$\tilde{\chi}_j^- \to \tilde{\nu}_\ell \ell^- \tag{3}$$

and

$$\tilde{\chi}_j^- \to W^-\tilde{\chi}_1^0 \ . \tag{4}$$

Then the momentum vector of $\ell = e, \mu, \tau$ or the W boson (if W decays hadronically) may be used to study the following triple products:

$$O_{\mathrm{T}}^3 = (\mathbf{p}_e \times \mathbf{p}_{\ell,W}) \cdot \mathbf{t}^{\pm} \ , \qquad O_{\mathrm{T}}^4 = (\mathbf{t}^+ \times \mathbf{t}^-) \cdot \mathbf{p}_{\ell,W} \ . \tag{5}$$

Possible T-odd observables based on the triple products in (2) and (5) would be

$$\langle O_{\mathrm{T}}^i \rangle \ , \quad \langle \mathrm{sgn}(O_{\mathrm{T}}^i) \rangle \ , \quad i = 1, \ldots, 4 \ . \tag{6}$$

However, as we will show below also the T-odd observables (6) vanish in Born approximation and neglecting terms proportional to the electron mass.

In the present paper we examine again the triple product correlations (2) and (5). We give a further argument why they have to vanish. In order to make use of the transverse beam polarization in chargino production we define the azimuthal asymmetries for the cases:
(i) azimuthal distribution of $\tilde{\chi}_j^-$ (when the direction of flight of the charginos can be reconstructed);
(ii) azimuthal distribution of the decay product in the decays (3) or (4). As we will demonstrate, the azimuthal asymmetry, though CP-even, may serve as a good observable to study the effects of CP phases.

This paper is organized as follows. In Sect. 2 we present the formulae for the cross section of (1) with transverse beam polarization and the decays (3) and (4). In Sect. 3 we argue why the T-odd observables in (6) vanish if only one of the subsequent chargino decays is considered. We define in Sect. 4 the azimuthal asymmetries and present our numerical results. Section 5 contains our conclusions.

2 Cross section

The Feynman diagrams contributing to the process (1) are given in Fig. 1. The relevant parts of the interaction Lagrangian which contribute to the process $e^+e^- \to \tilde{\chi}_i^+\tilde{\chi}_j^-$ and the subsequent two-body decays $\tilde{\chi}_j^- \to \tilde{\nu}_\ell \ell^-$ and $\tilde{\chi}_j^- \to W^-\tilde{\chi}_1^0$ are given by [1]

$$\mathcal{L}_{Ze^-e^+} = -\frac{g}{\cos\Theta_\mathrm{W}} Z_\mu \bar{\psi}_e \gamma^\mu (L_e P_L + R_e P_R)\psi_e \ , \quad (7)$$

$$\mathcal{L}_{Z\tilde{\chi}^+\tilde{\chi}^-} = \frac{g}{\cos\Theta_\mathrm{W}} Z_\mu \overline{\tilde{\chi}_i^+} \gamma^\mu (O'^L_{ij} P_L + O'^R_{ij} P_R)\tilde{\chi}_j^+ \ , \quad (8)$$

$$\mathcal{L}_{\ell\tilde{\nu}\tilde{\chi}^+} = -gV_{j1}^* \overline{\tilde{\chi}_j^{+C}} P_L \ell \tilde{\nu}^* + \mathrm{h.c.} \ , \quad (9)$$

$$\mathcal{L}_{W^-\tilde{\chi}^+\tilde{\chi}^0} = gW_\mu^- \overline{\tilde{\chi}_k^0} \gamma^\mu (O^L_{kj} P_L + O^R_{kj} P_R)\tilde{\chi}_j^+ + \mathrm{h.c.} \ , \quad (10)$$

with

$$O'^L_{ij} = -V_{i1}V_{j1}^* - \frac{1}{2}V_{i2}V_{j2}^* + \delta_{ij}\sin^2\Theta_\mathrm{W} \ , \quad (11)$$

$$O'^R_{ij} = -U_{i1}^*U_{j1} - \frac{1}{2}U_{i2}^*U_{j2} + \delta_{ij}\sin^2\Theta_\mathrm{W} \ , \quad (12)$$

and

$$O^L_{kj} = -\frac{1}{\sqrt{2}} N_{k4} V_{j2}^* + N_{k2} V_{j1}^* \ ,$$
$$O^R_{kj} = \frac{1}{\sqrt{2}} N_{k3}^* U_{j2} + N_{k2}^* U_{j1} \ , \quad (13)$$

where $L_e = -1/2 + \sin^2\Theta$, $R_e = \sin^2\Theta$, $P_{L,R} = 1/2(1 \mp \gamma_5)$, g is the weak coupling constant, $e = g\sin\Theta_\mathrm{W}$ and Θ_W is the Weinberg angle. The unitary 2×2 mixing matrices U and V diagonalize the chargino mass matrix $\mathcal{M}_C$, $U^*\mathcal{M}_C V^{-1} = \mathrm{diag}(m_{\chi_1}, m_{\chi_2})$. N_{ij} is the complex unitary 4×4 matrix which diagonalizes the neutral gaugino–higgsino mass matrix $Y_{\alpha\beta}$, $N_{i\alpha}^* Y_{\alpha\beta} N_{k\beta}^* = m_{\chi_i^0}\delta_{ik}$, in the basis $(\tilde{B}, \tilde{W}^3, \tilde{H}_1^0, \tilde{H}_2^0)$ [1].

For the calculation of the amplitude squared of the process (1) with subsequent decays (3) and (4), we use the spin-density matrix formalism [13,14]. The amplitude squared (without summing over the polarization of the charginos) can be written in the following way:

$$\rho_P^{\lambda_i\lambda_i'\lambda_j\lambda_j'} = \delta_{\lambda_i\lambda_i'}\delta_{\lambda_j\lambda_j'} P + \delta_{\lambda_j\lambda_j'} \sum_a \sigma^a_{\lambda_i\lambda_i'} \Sigma^a + \delta_{\lambda_i\lambda_i'} \sum_b \sigma^b_{\lambda_j\lambda_j'} \Sigma^b + \sum_{ab} \sigma^a_{\lambda_i\lambda_i'} \sigma^b_{\lambda_j\lambda_j'} \Sigma^{ab} \ , \quad (14)$$

where the coefficient P represents the part of the amplitude squared which is independent of the polarization of the $\tilde{\chi}^{\pm}$'s, and Σ^a and Σ^b contain the parts which depend on the polarization of $\tilde{\chi}_i^+$ and $\tilde{\chi}_j^-$, respectively. Finally Σ^{ab} contains the part which depends on the polarization of both $\tilde{\chi}^{\pm}$'s. In (14) $\sigma^{a,b}$ $(a,b = 1,2,3)$ denote the Pauli matrices and λ_i, λ_i' (λ_j, λ_j') are the helicity indices of $\tilde{\chi}_i^+$ $(\tilde{\chi}_j^-)$.

In the treatment of beam polarizations we use the general parametrization which, in the limit of vanishing electron mass, $m_e \to 0$, is given by

$$\lim_{m_e\to 0} \frac{1}{2}(\mathbf{1} + \gamma_5 \not{s}_{e^-})(\not{p}_1 + m_e) = \frac{1}{2}(\mathbf{1} + P_L\gamma_5 + \gamma_5 P_\mathrm{T} \not{t}^-)\not{p}_1 \quad (15)$$

and

$$\lim_{m_e\to 0} \frac{1}{2}(\mathbf{1} + \gamma_5 \not{s}_{e^+})(\not{p}_2 - m_e) = \frac{1}{2}(\mathbf{1} - \bar{P}_L\gamma_5 + \gamma_5 \bar{P}_\mathrm{T} \not{t}^+)\not{p}_2 \ , \quad (16)$$

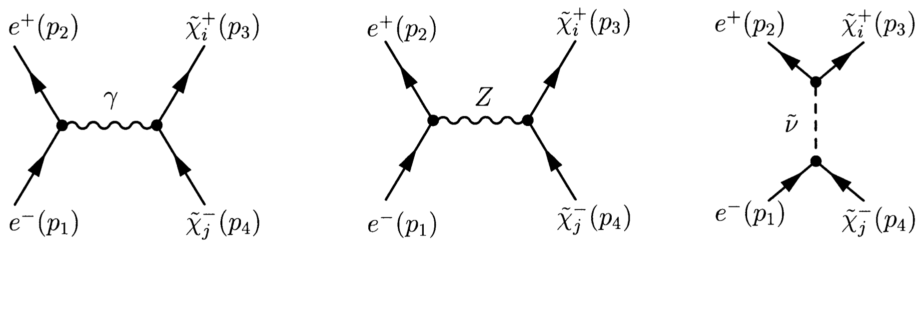

Fig. 1. Feynman diagrams for $e^+e^- \to \tilde{\chi}_i^+\tilde{\chi}_j^-$

where s_{e^-} (s_{e^+}) is the polarization vector and t^- (t^+) the transverse beam polarization 4-vector of e^- (e^+), respectively. In (15) (or (16)) P_L ($\bar{P}_L$) $[-1 \le P_L, \bar{P}_L \le 1]$ denotes the degree of the longitudinal polarization of e^- (e^+) and $P_{\rm T}$ ($\bar{P}_{\rm T}$) $[0 \le P_{\rm T}, \bar{P}_{\rm T} \le 1]$ denotes the degree of transverse polarization of e^- (e^+), statisfying $P_L^2 + P_{\rm T}^2 \le 1$ and $\bar{P}_L^2 + \bar{P}_{\rm T}^2 \le 1$. The P terms are given by

$$\begin{aligned} P = {} & P(\gamma\gamma) + P(\gamma\tilde{\nu}) + P(\gamma Z) \\ & + P(ZZ) + P(Z\tilde{\nu}) + P(\tilde{\nu}\tilde{\nu}) \ , \end{aligned} \tag{17}$$

where in the following we only list the part which involves the transverse beam polarization (the terms not dependent on the beam polarization or the terms which depend on the longitudinal beam polarization can be found in [15]):

$$P(\gamma\gamma) = P_{\rm T}\bar{P}_{\rm T}\ 2\delta_{ij} e^4 |\Delta(\gamma)|^2 (-r_1) \ , \tag{18}$$

$$\begin{aligned} P(\gamma\tilde{\nu}) = {} & -P_{\rm T}\bar{P}_{\rm T}\ \frac{1}{2}\delta_{ij} e^2 g^2 \Delta(\gamma)\Delta(\tilde{\nu})^* \\ & \times \mathrm{Re}\{V_{i1}^* V_{j1}(r_1 - r_2)\} \ , \end{aligned} \tag{19}$$

$$\begin{aligned} P(\gamma Z) = {} & P_{\rm T}\bar{P}_{\rm T}\ e^2 \delta_{ij} \frac{g^2}{\cos\Theta_W^2} \Delta(\gamma)\Delta(Z)^* \\ & \times \mathrm{Re}\{(O_{ij}^{\prime R} - O_{ij}^{\prime L}) \\ & \times [(L_e + R_e)(-r_1) + (L_e - R_e) r_2]\} \ , \end{aligned} \tag{20}$$

$$\begin{aligned} P(ZZ) = {} & P_{\rm T}\bar{P}_{\rm T}\ \frac{g^4}{\cos\Theta_W^4} |\Delta(Z)|^2 L_e R_e \\ & \times (|O_{ij}^{\prime L}|^2 + |O_{ij}^{\prime R}|^2)(-r_1) \ , \end{aligned} \tag{21}$$

$$\begin{aligned} P(Z\tilde{\nu}) = {} & -P_{\rm T}\bar{P}_{\rm T}\ \frac{1}{2}\frac{g^4}{\cos\Theta_W^2} \Delta(Z)\Delta(\tilde{\nu})^* R_e \\ & \times \mathrm{Re}\{V_{i1}^* V_{j1} O_{ij}^{\prime L}(r_1 - r_2)\} \ , \end{aligned} \tag{22}$$

$$P(\tilde{\nu}\tilde{\nu}) = 0 \ , \tag{23}$$

where we have introduced the shorthand notation

$$\begin{aligned} r_1 = {} & [(t^- p_4)(t^+ p_3) + (t^- p_3)(t^+ p_4)](p_1 p_2) \\ & + [(p_1 p_4)(p_2 p_3) + (p_1 p_3)(p_2 p_4) \\ & - (p_1 p_2)(p_3 p_4)](t^- t^+) \ , \end{aligned} \tag{24}$$

$$\begin{aligned} r_2 = {} & \mathrm{i}\ \varepsilon^{\mu\nu\rho\sigma} [t_\mu^+ p_{1\nu} p_{2\rho} p_{4\sigma} (t^- p_3) \\ & + t_\mu^- p_{1\nu} p_{2\rho} p_{3\sigma} (t^+ p_4) + t_\mu^- t_\nu^+ p_{2\rho} p_{4\sigma} (p_1 p_3) \\ & + t_\mu^- t_\nu^+ p_{1\rho} p_{3\sigma} (p_2 p_4)] \ , \end{aligned} \tag{25}$$

where $\Delta(Z) = \mathrm{i}/(s - m_Z^2), \Delta(\tilde{\nu}) = \mathrm{i}/(t - m_{\tilde{\nu}}^2)$, with $s = (p_1 + p_2)^2$, $t = (p_1 - p_4)^2$, $m_{\tilde{\nu}}$ (m_Z) is the mass of the sneutrino (Z boson) and $\varepsilon^{0123} = 1$. For the evaluation of the traces we have used the FeynCalc package [16]. Note that only terms bilinearly dependent on the transverse beam polarizations appear for $m_e \to 0$, since the couplings to e^+e^- are vector- or axial-vector-like [12,17,18] (for the $\tilde{\nu}$ exchange the coupling to e^+e^- can be brought to that form via Fierz identities). The cross section for the process (1) is given by

$$\mathrm{d}\sigma = \frac{1}{2(2\pi)^2} \frac{q}{s^{3/2}}\ P\ \mathrm{d}\cos\theta\ \mathrm{d}\phi \ , \tag{26}$$

where P contains the terms for arbitrary beam polarization and q is the momentum of the $\tilde{\chi}^{\pm}$'s.

Now we consider the $\Sigma^{a,b}$ terms, which means that we take into account the polarization (or equivalently the decay) of one of the two produced charginos. The Σ^a term is given by

$$\begin{aligned} \Sigma^a = {} & \Sigma^a(\gamma\gamma) + \Sigma^a(\gamma\tilde{\nu}) + \Sigma^a(\gamma Z) + \Sigma^a(ZZ) \\ & + \Sigma^a(Z\tilde{\nu}) + \Sigma^a(\tilde{\nu}\tilde{\nu}) \ , \end{aligned} \tag{27}$$

where in the following we again list only the part which involves the transverse beam polarization (for the terms independent of the beam polarization or the terms which depend on the longitudinal beam polarization see [15]):

$$\Sigma^a(\gamma\gamma) = 0 \ , \tag{28}$$

$$\begin{aligned} \Sigma^a(\gamma\tilde{\nu}) = {} & -P_{\rm T}\bar{P}_{\rm T}\ \frac{1}{2}\delta_{ij} e^2 g^2 \Delta(\gamma)\Delta(\tilde{\nu})^* \\ & \times \mathrm{Re}\{V_{i1}^* V_{j1}(r_1^a + r_2^a)\} \ , \end{aligned} \tag{29}$$

$$\begin{aligned} \Sigma^a(\gamma Z) = {} & P_{\rm T}\bar{P}_{\rm T}\ e^2 \delta_{ij} \frac{g^2}{\cos\Theta_W^2} \Delta(\gamma)\Delta(Z)^* \\ & \times \mathrm{Re}\{(O_{ij}^{\prime R} - O_{ij}^{\prime L})[(L_e + R_e) r_1^a \\ & + (L_e - R_e) r_2^a]\} \ , \end{aligned} \tag{30}$$

$$\begin{aligned} \Sigma^a(ZZ) = {} & P_{\rm T}\bar{P}_{\rm T}\ \frac{g^4}{\cos^4\Theta_{\rm W}} \\ & \times |\Delta(Z)|^2 L_e R_e (|O_{ij}^{\prime R}|^2 - |O_{ij}^{\prime L}|^2) r_1^a \ , \end{aligned} \tag{31}$$

$$\begin{aligned} \Sigma^a(Z\tilde{\nu}) = {} & -P_{\rm T}\bar{P}_{\rm T}\ \frac{g^4}{2\cos^2\Theta_{\rm W}} \Delta(Z)\Delta(\tilde{\nu})^* R_e \\ & \times \mathrm{Re}\{V_{i1}^* V_{j1} O_{ij}^{\prime L}(r_1^a + r_2^a)\} \ , \end{aligned} \tag{32}$$

$$\Sigma^a(\tilde{\nu}\tilde{\nu}) = 0 \ , \tag{33}$$

with

$$\begin{aligned} r_1^a = {} & -m_{\chi_i}\{[(t^+ p_4)(s^a t^-) + (s^a t^+)(t^- p_4)](p_1 p_2) \\ & + [(s^a p_2)(p_1 p_4) + (s^a p_1)(p_2 p_4) \\ & - (s^a p_4)(p_1 p_2)](t^- t^+)\} \end{aligned} \tag{34}$$

$$\begin{aligned} r_2^a = {} & \mathrm{i}\ \varepsilon^{\mu\nu\rho\sigma}\ m_{\chi_i} [t_\mu^+ p_{1\nu} p_{2\rho} p_{4\sigma} (s^a t^-) \\ & + t_\mu^- t_\nu^+ p_{2\rho} p_{4\sigma} (s^a p_1) - s_\mu^a t_\nu^- p_{1\rho} p_{2\sigma} (t^+ p_4) \\ & - s_\mu^a t_\nu^- t_\rho^+ p_{1\sigma} (p_2 p_4)] \ , \end{aligned} \tag{35}$$

where the polarization basis 4-vectors s^a ($a = 1,2,3$) for $\tilde{\chi}_i^+$ fulfill the orthogonality relations $s^a \cdot s^c = -\delta^{ac}$ and $s^a \cdot p_3 = 0$. Σ^b is obtained by making the replacements $s^a \to -s^b, m_{\chi_i} \to m_{\chi_j}, p_4 \to p_3$ in (24) and (34).

The spin-density matrices for the decays $\tilde{\chi}_j^- \to \tilde{\nu}_\ell \ell^-$ (3) and $\tilde{\chi}_j^- \to \tilde{\chi}_1^0 W^-$ (4) can be written as

$$(\rho_D)_{\lambda_j^\prime \lambda_j} = D\delta_{\lambda_j^\prime \lambda_j} + \Sigma_D^a \sigma_{\lambda_j^\prime \lambda_j}^a \ , \tag{36}$$

where the expansion coefficients are

$$D(\tilde{\nu}\ \ell) = \frac{g^2}{2} |V_{j1}|^2 (m_{\chi_j}^2 - m_{\tilde{\nu}}^2) \ , \tag{37}$$

$$\Sigma_D^b(\tilde{\nu}\ \ell) = g^2 |V_{j1}|^2 m_{\chi_j} (s^b \cdot p_\ell) \ , \tag{38}$$

for the decay (3) and

$$
\begin{aligned}
D(\tilde{\chi}_1^0\, W) &= g^2(|O_{1j}^L|^2 + |O_{1j}^R|^2) \\
&\times \left[\frac{(m_{\chi_1^0}^2 + m_{\chi_j}^2) m_W^2 + (m_{\chi_1^0}^2 - m_{\chi_j}^2)^2 - 2 m_W^4}{2 m_W^2} \right] \\
&- 6 g^2 \mathrm{Re}(O_{1j}^{L\,*} O_{1j}^R) m_{\chi_1^0} m_{\chi_j} \ , \qquad (39)\\
\Sigma_D^b(\tilde{\chi}_1^0\, W) &= g^2(|O_{1j}^L|^2 - |O_{1j}^R|^2) m_{\chi_j} \\
&\times \left[\frac{m_{\chi_j}^2 - m_{\chi_1^0}^2 - 2 m_W^2}{m_W^2} \right] (s^b \cdot p_W) \ , \qquad (40)
\end{aligned}
$$

for the decay (4). Using (14) and summing over the polarization of $\tilde{\chi}_i^+$, whose decay is not considered, finally gives the cross section for $e^+e^- \to \tilde{\chi}_i^+ \tilde{\chi}_j^- \to \tilde{\chi}_i^+ \ell^- \tilde{\nu}_\ell$ $(\tilde{\chi}_i^+ W^- \tilde{\chi}_1^0)$:

$$
\mathrm{d}\sigma = \frac{2}{s} \left[P D + \Sigma_P^a \Sigma_D^a \right] |\Delta(\tilde{\chi}_j^-)|^2 \mathrm{dLips} \ , \qquad (41)
$$

where P and Σ_P^a involve the terms for arbitrary beam polarization. The Lorentz invariant phase space element dLips is given in Appendix B for the two decays (3) and (4).

3 Triple product correlations with transverse beam polarization

In the following we argue why T-odd observables as in (6) based on triple product correlations of the sort as in (2) and in (5) are expected to vanish at tree-level if at least one subsequent chargino decay is not observed.

We discuss first the T-odd observables $\langle O_{\mathrm{T}}^{1,2} \rangle$ based on the triple product correlations in (2), which involve only the production cross section $\sigma(e^+e^- \to \tilde{\chi}_i^+ \tilde{\chi}_j^-)$ and the quantity P, (17). First we note that these triple products are contained only in the kinematic quantity r_2 in (25). We note further that the two produced charginos should be different mass eigenstates, i.e. $i \neq j$; otherwise the prefactor $V_{i1}^* V_{j1} O_{ij}^{\prime L}$ in (22) would be real. Then the γ exchange does not contribute. However, it can be shown that also for $i \neq j$ the prefactor $V_{i1}^* V_{j1} O_{ij}^{\prime L}$ in (22) is real. In fact, this can be verified by a short calculation using the parametrization for V [6,19]

$$
V = \begin{pmatrix} \cos\theta_2 & \mathrm{e}^{-\mathrm{i}\phi_2} \sin\theta_2 \\ -\mathrm{e}^{\mathrm{i}\phi_2} \sin\theta_2 & \cos\theta_2 \end{pmatrix} . \qquad (42)
$$

This result can also be deduced from the formulae given in [7]. The reason behind lies in the CP property of the quantity r_2. We adopt the method of [12] to examine the behavior of r_2 under a CP transfomation. We first choose the transverse polarizations of e^- and e^+ either parallel or anti-parallel to each other ($\mathbf{t}^- = \mathbf{t}^+$ or $\mathbf{t}^- = -\mathbf{t}^+$). Then the last two terms in (25) are identical zero. Applying a CP transformation (in the CM system) to the first two terms of r_2 as follows:

$$
\begin{aligned}
\mathbf{t}^+(\mathbf{p}_1 \times \mathbf{p}_4)(\mathbf{t}^- \cdot \mathbf{p}_3) &\xrightarrow{C} \mathbf{t}^-(\mathbf{p}_2 \times \mathbf{p}_4)(\mathbf{t}^+ \cdot \mathbf{p}_3) \\
&\xrightarrow{P} -\mathbf{t}^-(\mathbf{p}_2 \times \mathbf{p}_4)(\mathbf{t}^+ \cdot \mathbf{p}_3), \qquad (43)
\end{aligned}
$$

one finds that r_2 is CP-even (here we sum over the charges of the final charginos so that $\mathbf{p}_3 \xrightarrow{C} \mathbf{p}_3$ and $\mathbf{p}_4 \xrightarrow{C} \mathbf{p}_4$). On the other hand r_2 is T-odd, where T stands for the so-called naive time reversal (i.e. all momentum and polarization vectors are reversed without interchanging initial and final state). Therefore, the prefactor of r_2 in (25) vanishes as a consequence of CPT. The same conclusion can be derived for the case that the transverse polarizations of e^- and e^+ are orthogonal to each other. Non-zero contributions to the T-odd observables $\langle O_{\mathrm{T}}^{1,2} \rangle$ may arise if terms of the order $\mathcal{O}(m_e)$ are included.

As the next step we discuss the triple product correlations $O_{\mathrm{T}}^{3,4}$, which means we take into account also the subsequent decay of one of the charginos. In this case we have to include the terms Σ^a, (28)–(33), which depend on the polarization of the decaying chargino. As can be seen, the quantity r_2^a of $\Sigma^a(Z\tilde{\nu})$ in (32) is the only term which contains the triple product correlations (5). However, its prefactor is again $\mathrm{Im}(V_{i1}^* V_{j1} O_{ij}^{\prime L})$, which is zero as shown above. Unlike in the previous case the reason for the vanishing prefactor is not a direct consequence of CPT. In fact, applying a CP transformation to r_2^a in the same manner as in the previous case shows that r_2^a is CP-odd. It is also T-odd.

Also in the present case there may be non-zero contributions to the T-odd observables in (6) proportional to m_e. In general non-zero contributions to the T-odd observables based on the triple products in (2) and (5) may also arise by the inclusion of one-loop contributions.

Thus we have to conclude that only the Σ^{ab} terms contain non-vanishing triple product correlations with transverse beam polarization. In order to measure observables based on such triple product correlations the decays of both $\tilde{\chi}^{\pm}$'s must be taken into account [20]. However, in this case transversely polarized beams are not really necessary, because the same combinations of CP-violating couplings appear in Σ^{ab} already in the case of unpolarized beams [21].

Although we have derived our results within the MSSM, we would like to point out that our conclusions remain valid if SUSY flavor violating terms are included. In such a case the Lagrange density in (9) is modified, however, possible CP-violating phases from the flavor violating sector drop out in the amplitude for the $\tilde{\nu}$ exchange (in this context see also [22]). This is interesting since in this case φ_μ may not be restricted due to the electron electric dipole moment (EDM) [23].

4 Azimuthal asymmetry

As we have seen transverse beam polarization does not lead to a T-odd (CP-odd) observable if chargino production and the decay of only one of the charginos is considered. In order to measure the CP-violating parameter φ_μ and the phase of the $U(1)$ gaugino mass parameter φ_{M_1} in

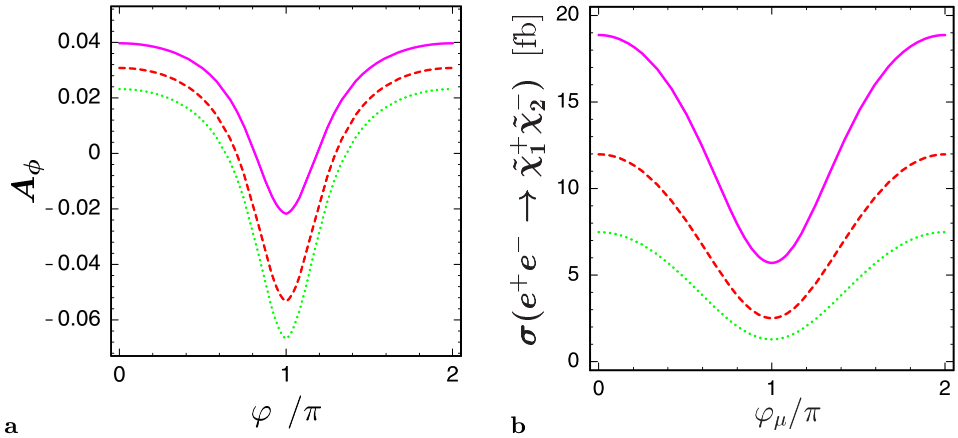

Fig. 2a,b. The azimuthal asymmetry in (46) and the cross section $\sigma(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-)$ as a function of φ_μ. The three lines correspond to values of $|\mu|$ (from the top to the bottom) of $|\mu| = (300, 350, 400)$ GeV. The other parameters are chosen as $M_2 = 200$ GeV, $\tan\beta = 3$, $\sqrt{s} = 800$ GeV and $m_{\tilde{\nu}} = 400$ GeV

the reaction $e^+e^- \to \tilde{\chi}_i^+\tilde{\chi}_j^-$ with transverse beam polarization we propose as observable an azimuthal asymmetry analogous to that of [7,8]

$$A_\phi = \frac{\int^+ (\mathrm{d}\sigma/\mathrm{d}\phi)\mathrm{d}\phi - \int^- (\mathrm{d}\sigma/\mathrm{d}\phi)\mathrm{d}\phi}{\sigma}\ , \qquad (44)$$

where ϕ is the azimuthal angle of the $\tilde{\chi}^{\pm}$'s. In (44) $\int^{\pm}$ corresponds to an integration over regions where $\cos 2\phi$ (or $\sin 2\phi$) is positive or negative. The integration in the numerator has the effect of projecting out the terms $\propto P_{\mathrm{T}}\bar{P}_{\mathrm{T}}$ in the formulae for the differential cross section $\mathrm{d}\sigma/\mathrm{d}\phi$. Choosing the beam direction along the z-axis and the transverse polarization of e^- along the x-axis (see Appendix A), the kinematical factor in (24) can be rewritten as

$$r_1 = -\frac{1}{2}q^2 s\ \sin^2\theta(\sin 2\phi \sin\bar{\alpha} + \cos 2\phi\cos\bar{\alpha})\ , \qquad (45)$$

where $\bar{\alpha}$ is the angle between the transverse polarization vectors of e^- and e^+, and the other quantities are defined in the Appendix A. This means for the azimuthal asymmetry that we have two possible integrations depending on how the two transverse beam polarizations are orientated to each other. For $\bar{\alpha} = \pi/2$ (44) leads to

$$A_\phi = \frac{1}{\sigma}\left[\int_0^{\pi/2} - \int_{\pi/2}^{\pi} + \int_{\pi}^{3\pi/2} - \int_{3\pi/2}^{2\pi}\right]\frac{\mathrm{d}\sigma}{\mathrm{d}\phi}\mathrm{d}\phi\ . \qquad (46)$$

If we had chosen $\bar{\alpha} = 0, \pi$ instead, the integration over ϕ would be in steps of $\pi/4$. Under favorable conditions the momentum of the $\tilde{\chi}^{\pm}$'s can be reconstructed. For such a case we calculate A_ϕ and σ for $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-$ as a function of $\varphi_\mu \in [0, 2\pi]$ and the choices of $|\mu| = (300, 350, 400)$ GeV, fixing the other parameters as $M_2 = 200$ GeV, $\tan\beta = 3$, $m_{\tilde{\nu}} = 400$ GeV for $\sqrt{s} = 800$ GeV. We assume that the same degree of transverse polarization is feasible as for the longitudinal polarization, this means we take $P_{\mathrm{T}} = 80\%$ and $\bar{P}_{\mathrm{T}} = 60\%$. Figure 2 shows the result. As can be seen A_ϕ depends quite strongly on φ_μ. We have found that this dependence gets much weaker for increasing $\tan\beta$, since in the limit $\tan\beta \to \infty$ the mixing angles and mass eigenvalues in the chargino sector are independent of φ_μ. We have compared the phase dependence of the cross section with the numerical results of [22] and found agreement.

We have also studied the reaction $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$, and we have found the φ_μ dependence is much weaker. The reason is that the φ_μ dependence of the denominator and numerator in (46) almost cancel each other in a large part of the MSSM parameter space.

The reconstruction of the direction of the $\tilde{\chi}^{\pm}$'s is not necessary if we consider the subsequent decays $\tilde{\chi}_j^- \to \tilde{\nu}_\ell \ell^-$, (3), or $\tilde{\chi}_j^- \to W^-\tilde{\chi}_1^0$, (4), and the corresponding azimuthal distribution of ℓ^- or W^-. We define the azimuthal asymmetry according to (46) with the cross section given in (41). Note that only the terms (18)–(23) and (28)–(33) together with the phase space elements (which are defined in the appendices) depend on the azimuthal angle of ℓ^- or W^-.

In the following calculations of A_ϕ we assume $P_{\mathrm{T}} = 80\%$ and $\bar{P}_{\mathrm{T}} = 60\%$. In Fig. 3 we show the azimuthal asymmetry, (46), of ℓ^- and W^- as a function of $\varphi_\mu \in [0, 2\pi]$. The MSSM parameters are chosen to be $|\mu| = 400$ GeV, $M_2 = 200$ GeV, $\tan\beta = 3$, $\varphi_{M_1} = 0$, $m_{\tilde{\nu}} = 150$ GeV and we will assume the GUT relation $|M_1| = (5/3)\tan^2\theta_{\mathrm{W}}\ M_2$

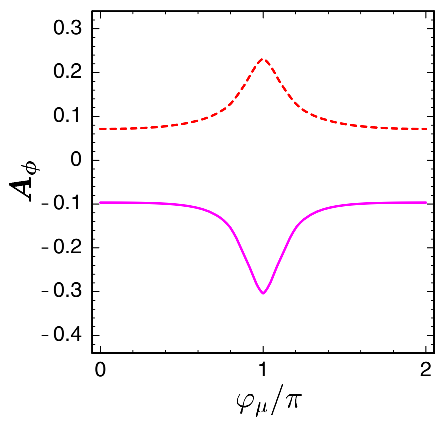

Fig. 3. The azimuthal asymmetry A_ϕ, (46), for the reaction $\sigma(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-)$ at $\sqrt{s} = 800$ GeV, with subsequent decays $\tilde{\chi}_2^- \to \tilde{\nu}_\ell \ell^-$ (solid line) and $\tilde{\chi}_2^- \to W^-\tilde{\chi}_1^0$ (dashed line) as a function of φ_μ. The other parameters are $|\mu| = 400$ GeV, $M_2 = 200$ GeV, $\tan\beta = 3$, $\varphi_{M_1} = 0$, $m_{\tilde{\nu}} = 150$ GeV

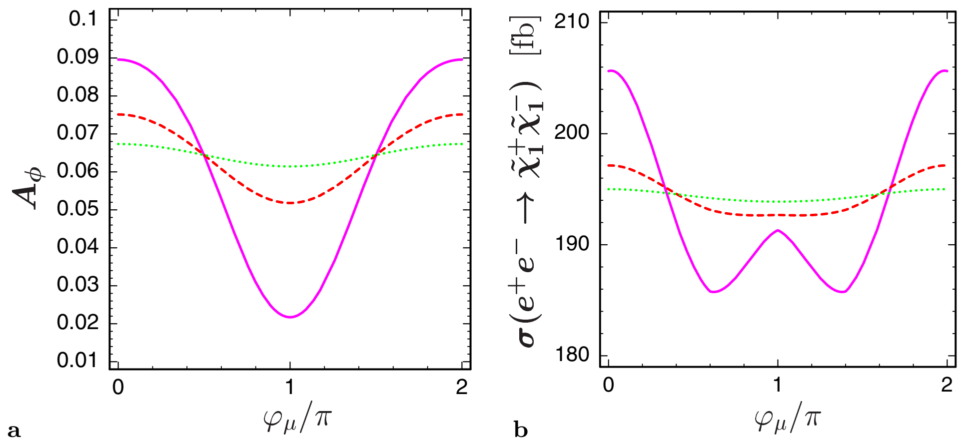

Fig. 4. a The azimuthal asymmetry A_ϕ, (46), for $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$, $\tilde{\chi}_1^- \to \tilde{\nu}_\ell \ell^-$ and **b** the cross section for $\sigma(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-)$ as a function of φ_μ. The three lines correspond to $\tan\beta = 3$ (solid line), 10 (dashed line), 40 (dotted line), with $|\mu| = 300\,\mathrm{GeV}$, $M_2 = 200\,\mathrm{GeV}$ and $m_{\tilde{\nu}} = 150\,\mathrm{GeV}$. The CM energy is taken to be $\sqrt{s} = 500\,\mathrm{GeV}$

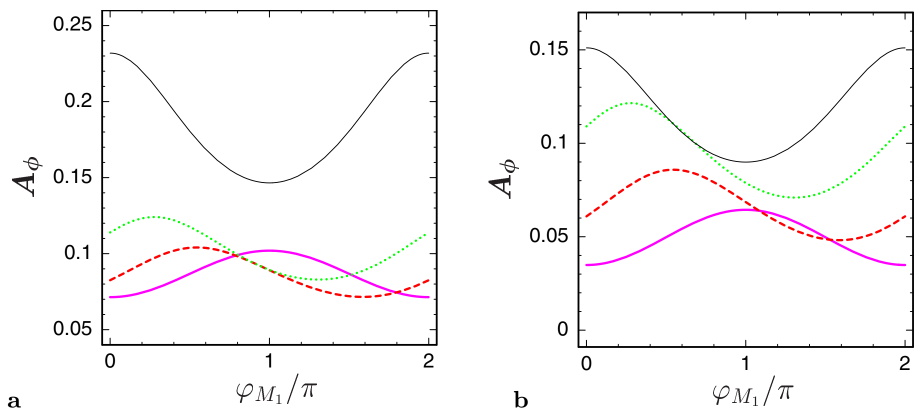

Fig. 5. The azimuthal asymmetry A_ϕ, (46), for the reaction $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-$, $\tilde{\chi}_2^- \to \tilde{\chi}_1^0 W^-$ as a function of φ_{M_1} for $\sqrt{s} = 800\,\mathrm{GeV}$, $|\mu| = 400\,\mathrm{GeV}$, $M_2 = 200\,\mathrm{GeV}$, $\tan\beta = 3$, $\varphi_\mu = 0$ (thick solid line), $\pi/2$ (dashed line), $3\pi/4$ (dotted line), π (thin solid line). **a** shows A_ϕ for $m_{\tilde{\nu}} = 150\,\mathrm{GeV}$ and **b** shows A_ϕ for $m_{\tilde{\nu}} = 2\,\mathrm{TeV}$

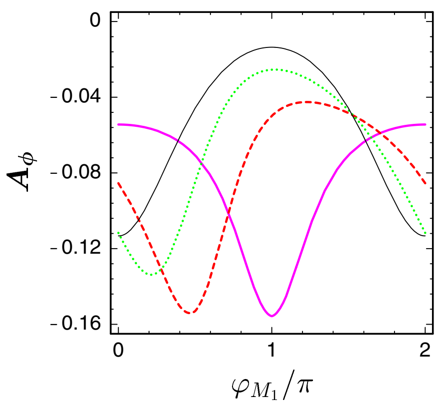

Fig. 6. The azimuthal asymmetry A_ϕ, (46), for the reaction $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$ ($\sqrt{s} = 800\,\mathrm{GeV}$) with subsequent decay $\tilde{\chi}_1^- \to \tilde{\chi}_1^0 W^-$ as a function of φ_{M_1}. We fix $|\mu| = 400\,\mathrm{GeV}$, $M_2 = 400\,\mathrm{GeV}$, $\tan\beta = 3$, $m_{\tilde{\nu}} = 150\,\mathrm{GeV}$, $\varphi_\mu = 0$ (thick solid line), $\pi/2$ (dashed line), $3\pi/4$ (dotted line), π (thin solid line)

throughout. We vary φ_μ over the whole range altough it may in general be restricted due to the EDM measurements. As can be seen in Fig. 3 the CP-conserving values $\varphi_\mu = 0$ and $\varphi_\mu = \pi$ give quite different results for A_ϕ. The corresponding cross section is $\sigma(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-) \approx 29\,\mathrm{fb}$ for $\varphi_\mu = 0$ and decreases monotonically for increasing φ_μ until $\sigma(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-) \approx 1.7\,\mathrm{fb}$ for $\varphi_\mu = \pi$.

In Fig. 4 we display A_ϕ and the cross section for the reaction $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$, $\tilde{\chi}_1^- \to \tilde{\nu}_\ell \ell^-$, as a function of φ_μ for three values of $\tan\beta = (3, 10, 40)$, taking $|\mu| = 300\,\mathrm{GeV}$, $M_2 = 200\,\mathrm{GeV}$ and $m_{\tilde{\nu}} = 150\,\mathrm{GeV}$. As can be seen the variations of A_ϕ (Fig. 4a) and $\sigma(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-)$ (Fig. 4b) decrease with increasing $\tan\beta$. For low $\tan\beta$ (= 3) again the CP-conserving points lead to quite different results for A_ϕ. Also the cross section depends in a significant way on φ_μ, for example, for $\tan\beta = 3$ the absolute minimum of the cross section is reached for CP-violating points $\varphi_\mu \approx \frac{3}{5}\pi, \frac{7}{5}\pi$ (see Fig. 4b).

In Fig. 5 we plot A_ϕ for the reaction $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-$, with $\tilde{\chi}_2^- \to W^-\tilde{\chi}_1^0$. We fix $|\mu| = 400\,\mathrm{GeV}$, $M_2 = 200\,\mathrm{GeV}$, $\tan\beta = 3$ and vary $\varphi_{M_1} \in [0, 2\pi]$ for $\varphi_\mu = (0, \pi/2, 3\pi/4, \pi)$. Figure 5a shows A_ϕ for $m_{\tilde{\nu}} = 150\,\mathrm{GeV}$ and Fig. 5b for $m_{\tilde{\nu}} = 2\,\mathrm{TeV}$. For $m_{\tilde{\nu}} = 2\,\mathrm{TeV}$ (and assuming that the other sfermion masses of the first two generations are also heavy) the restriction from the EDMs on φ_μ is relaxed. One sees that A_ϕ depends quite strongly on the CP-violating phases φ_{M_1} and φ_μ. Note that the φ_{M_1} dependence is due to the decay amplitude.

In Fig. 6 we plot A_ϕ for the reaction $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$ with the subsequent decay $\tilde{\chi}_1^- \to \tilde{\chi}_1^0 W^-$. The MSSM parameters are chosen to be $|\mu| = 400\,\mathrm{GeV}$, $M_2 = 400\,\mathrm{GeV}$, $\tan\beta = 3$. We vary $\varphi_{M_1} \in [0, 2\pi]$ for $\varphi_\mu = (0, \pi/2, 3\pi/4, \pi)$. As can be seen also in this case the phase dependence of the azimuthal asymmetry of the W boson is very pronounced.

5 Conclusion

We have considered the process $e^+e^- \to \tilde{\chi}_i^+ \tilde{\chi}_j^-$ at a linear collider with transversely polarized e^+ and e^- beams. We have given the analytical expressions for the cross section of these processes in the spin-density matrix formalism. We have given arguments why triple product correlations involving the transverse $e^{\pm}$ polarizations vanish if at least one subsequent chargino decay is not observed. Our framework has been the MSSM, but this statement is also valid for the general MSSM with SUSY flavor violation. We have proposed and studied azimuthal asymmetries in the processes $e^+e^- \to \tilde{\chi}_i^+ \tilde{\chi}_j^- \to \tilde{\chi}_i^+ \ell^- \tilde{\nu}_\ell$ $(\tilde{\chi}_i^+ W^- \tilde{\chi}_1^0)$. We have demonstrated that these azimuthal asymmetries are well suited to investigate the effect of the SUSY CP phases φ_μ and φ_{M_1}.

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Appendix

A Momentum and polarization vectors

We define the transverse beam polarization 4-vectors in (15) and (16) as $t^- = \cos\alpha n_1 + \sin\alpha n_2$ and $t^+ = \cos\bar{\alpha} n_1 + \sin\bar{\alpha} n_2$. We choose the z-axis along the beam direction in the CM system, and $n_1 = (0,1,0,0)$, $n_2 = (0,0,1,0)$. Without loss of generality, we take $\alpha = 0$ throughout. The 4-momenta of the $\tilde{\chi}^{\pm}$'s are given by

$$p_{\chi_j} = p_4 = q(E_{\chi_j}/q, \cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta) \ , \quad (47)$$

with

$$E_{\chi_{i,j}} = \frac{s + m^2_{\chi_{i,j}} - m^2_{\chi_{j,i}}}{2\sqrt{s}} \ ,$$
$$q = \frac{\lambda^{\frac{1}{2}}(s, m^2_{\chi_i}, m^2_{\chi_j})}{2\sqrt{s}} \ , \quad (48)$$

where $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$. The three spin basis vectors of $\tilde{\chi}_j^-$ are chosen to be

$$\begin{aligned} s^1_{\chi_j} &= \left(0, \frac{\mathbf{s}_2 \times \mathbf{s}_3}{|\mathbf{s}_2 \times \mathbf{s}_3|}\right) \\ &= (0, \cos\phi\cos\theta, \sin\phi\cos\theta, -\sin\theta) \ , \\ s^2_{\chi_j} &= \left(0, \frac{\mathbf{p}_{e^-} \times \mathbf{p}_{\chi_j}}{|\mathbf{p}_{e^-} \times \mathbf{p}_{\chi_j}|}\right) = (0, -\sin\phi, \cos\phi, 0) \ , \\ s^3_{\chi_j} &= \frac{1}{m_{\chi_j}}\left(q, \frac{E_{\chi_j}}{q}\mathbf{p}_{\chi_j}\right) \\ &= \frac{E_{\chi_j}}{m_{\chi_j}}(q/E_{\chi_j}, \cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta) \ . \end{aligned} \quad (49)$$

The 4-momentum of the lepton in the decay $\tilde{\chi}_j^- \to \tilde{\nu}_\ell \ell^-$ is given by

$$p_\ell = |\mathbf{p}_\ell|(1, \cos\phi_1\sin\theta_1, \sin\phi_1\sin\theta_1, \cos\theta_1) \ , \quad (50)$$

where

$$|\mathbf{p}_\ell| = \frac{m^2_{\chi_j} - m^2_{\tilde{\nu}}}{2(E_{\chi_j} - q\cos\vartheta)} \ , \quad (51)$$

and

$$\cos\vartheta = \sin\theta\sin\theta_1\cos(\phi - \phi_1) + \cos\theta\cos\theta_1 \ . \quad (52)$$

The 4-momentum of the W in the decay $\tilde{\chi}_j^- \to \tilde{\chi}_1^0 W^-$ is given by

$$\begin{aligned} p_W = (&E_W, |\mathbf{p}_W|\cos\phi_1\sin\theta_1, \\ &|\mathbf{p}_W|\sin\phi_1\sin\theta_1, |\mathbf{p}_W|\cos\theta_1) \ . \end{aligned} \quad (53)$$

with

$$\begin{aligned} |\mathbf{p}_W^{\pm}| = &\left[2|\mathbf{p}_{\chi_j}|^2(1 - \cos^2\vartheta) + 2m^2_{\chi_j}\right]^{-1} \\ &\times \Big[(m^2_{\chi_j} + m^2_W - m^2_{\chi_1^0})|\mathbf{p}_{\chi_j}|\cos\vartheta \\ &\pm E_{\chi_j}\sqrt{\lambda(m^2_{\chi_j}, m^2_W, m^2_{\chi_1^0}) - 4|\mathbf{p}_{\chi_j}|^2\ m^2_W\ (1 - \cos^2\vartheta)}\Big] \ . \end{aligned} \quad (54)$$

There are two solutions $|\mathbf{p}_W^{\pm}|$ if $|\mathbf{p}^0_{\chi_j}| < |\mathbf{p}_{\chi_j}|$, where $|\mathbf{p}^0_{\chi_j}| = \lambda^{\frac{1}{2}}(m^2_{\chi_j}, m^2_W, m^2_{\chi_1^0})/2m_W$ is the chargino momentum if the W boson is produced at rest. The W decay angle ϑ is constrained in that case and the maximal angle $\vartheta^{\max}$ is given by

$$\sin\vartheta^{\max} = \frac{|\mathbf{p}^0_{\chi_j}|}{|\mathbf{p}_{\chi_j}|} = \frac{\sqrt{s}}{m_W}\frac{\lambda^{\frac{1}{2}}(m^2_{\chi_j}, m^2_W, m^2_{\chi_1^0})}{\lambda^{\frac{1}{2}}(s, m^2_{\chi_i}, m^2_{\chi_j})} \le 1 \ . \quad (55)$$

If $|\mathbf{p}^0_{\chi_j}| > |\mathbf{p}_{\chi_j}|$, the decay angle ϑ is not constrained and there is only the physical solution $|\mathbf{p}_W^+|$.

B Phase space

The Lorentz invariant phase space element in (41) is given by

$$\mathrm{dLips} = \frac{1}{2\pi}\mathrm{dLips}(s, p_{\chi_i}, p_{\chi_j})\mathrm{d}s_{\chi_j}\mathrm{dLips}(s_{\chi_j}, p_{\tilde{\nu}}, p_\ell) \quad (56)$$

for the subsequent decay $\tilde{\chi}_j^- \to \tilde{\nu}_\ell \ell^-$ (3), and by

$$\mathrm{dLips} = \frac{1}{2\pi} \mathrm{dLips}(s, p_{\chi_i}, p_{\chi_j}) \mathrm{d}s_{\chi_j} \times \sum_{\pm} \mathrm{dLips}(s_{\chi_j}, p_{\chi_1^0}, p_W^{\pm}) \tag{57}$$

for the subsequent decay $\tilde{\chi}_j^- \to \tilde{\chi}_1^0 W^-$ (4). The Lorentz invariant phase space elements in (56) and (57) read

$$\mathrm{dLips}(s, p_{\chi_i}, p_{\chi_j}) = \frac{1}{4(2\pi)^2} \frac{q}{\sqrt{s}} \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi, \tag{58}$$

$$\mathrm{dLips}(s_{\chi_j}, p_{\tilde{\nu}}, p_\ell) = \frac{1}{2(2\pi)^2} \frac{|\mathbf{p}_\ell|^2}{m_{\chi_j}^2 - m_{\tilde{\nu}}^2} \sin\theta_1 \, \mathrm{d}\theta_1 \, \mathrm{d}\phi_1 \, , \tag{59}$$

$$\mathrm{dLips}(s_{\chi_j}, p_{\chi_1^0}, p_W^{\pm}) = \frac{1}{4(2\pi)^2} \times \frac{|\mathbf{p}_W^{\pm}|^2}{|E_W^{\pm} \, |\mathbf{p}_{\chi_j}| \cos\vartheta - E_{\chi_j} \, |\mathbf{p}_W^{\pm}||} \sin\theta_1 \, \mathrm{d}\theta_1 \, \mathrm{d}\phi_1 \, . \tag{60}$$

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